Temperature-dependent anomalous statistics

1992 J. Phys. A: Math. Gen. 25 L245

(http://iopscience.iop.org/0305-4470/25/5/007)

J. Phys. A: Math. Gen. 25 (1992) L245–L248. Printed in the UK

LETTER TO THE EDITOR

Temperature-dependent anomalous statistics

Ashok Das† and Sudhakar Panda
International Centre for Theoretical Physics, 34100 Trieste, Italy

Received 11 November 1991

Abstract. We show that the anomalous statistics which arises in (2+1)-dimensional Chern–Simons gauge theories can become temperature dependent in the most natural way. We analyse and show that a statistics-changing phase transition can happen in these theories only as $T \to \infty$.

It is by now well known that, in 2+1 dimensions, a Chern–Simons gauge theory coupled to a matter current can lead to anomalous statistics [1] (for reviews of this subject see [2]). Such a phenomenon is interesting and can have physical applications [3]. In fact, it has been suggested that the phenomenon of high T_c superconductivity may have its origin in such theories [4] (for a review see [5]). It is, therefore, quite appropriate to study the behaviour of such theories at finite temperature. Among many questions of interest, one can ask, for example, if statistics can become temperature dependent in such theories and if so, whether one can envision a phase transition involving the two phases of normal statistics and anomalous statistics. We would like to point out here that Wen and Zee [6] had already discussed the possibility of such a phase transition which will arise from the restoration of a spontaneously broken gauge symmetry. Our analysis, however, is quite general in the absence of gauge symmetry breaking and is complementary to their analysis. We find that, in such theories, angular momentum and, therefore, statistics does, indeed, become temperature dependent and that the only phase transition that occurs in such a case is at $T \to \infty$.

Let us recapitulate very briefly how anomalous statistics arises in a Chern–Simons gauge theory. Let us consider a Dirac fermion, in 2+1 dimensions, interacting with an Abelian gauge field where the action for the gauge field is a pure Chern–Simons term [1]. (One can, of course, discuss anomalous statistics also in the context of a bosonic theory [7].) The Lagrangian density of such a system can be written as

$$\mathscr{L} = \frac{\kappa}{8\pi}\,\varepsilon^{\mu\nu\lambda}A_\mu(x)\partial_\nu A_\lambda(x) + \bar{\psi}(x)(\mathrm{i}\partial\!\!\!/ - m + gA\!\!\!/(x))\psi(x). \tag{1}$$

Here κ is a constant and in our convention, $\eta^{\mu\nu} = (+,-,-)$. It is important to note that in this theory, the Maxwell equations give

$$\varepsilon^{\mu\nu\lambda}\partial_\nu A_\lambda = \frac{4\pi g}{\kappa}\,\bar{\psi}\gamma^\mu\psi = \frac{4\pi g}{\kappa}\,j^\mu \tag{2}$$

† Permanent address: Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627, USA.

which allows us to express the gauge potentials explicitly in terms of the current. It is straightforward to calculate the angular momentum associated with (1) and, in the Coulomb gauge, it can be shown to have the form [1, 2, 8]

$$J=\int \mathrm{d}^2x[\mathrm{i}\varepsilon^{ij}x_i\psi^\dagger\partial_j\psi+\tfrac{1}{2}\psi^\dagger\sigma_3\psi]+\frac{g^2}{\kappa}Q^2 \tag{3}$$

where

$$Q=\int \mathrm{d}^2x\,j^0. \tag{4}$$

In writing (3), we have used the solutions of (2). Note that it is the last Q-dependent term in (3) which gives rise to an anomalous angular momentum in the present case [8]. With standard arguments, one can then show that the exchange of two fermions, in such a case, will lead to an additional phase factor, $\exp(2\pi\mathrm{i}g^2/\kappa)$ (see, for example, [6, 8] and De Sousa Gerbert in [2]). This is the origin of anomalous (fractional) statistics. We want to emphasize that it arises primarily from a non-vanishing κ and g and is completely determined by κ/g^2. Note, in particular, that a vanishing κ will necessarily force the theory into a zero current sector where the anomalous part of the angular momentum operator will vanish resulting in normal statistics.

To analyse the properties of such a system at finite temperature, it is useful to reformulate the theory in an alternate way. Let us consider the following system of fermions interacting with an Abelian gauge field:

$$\tilde{\mathscr{L}}=\bar{\psi}(\mathrm{i}\partial\!\!\!/-m+gA\!\!\!/)\psi+\bar{\Psi}_i(\mathrm{i}\partial\!\!\!/-M+\tilde{g}A\!\!\!/)\Psi_i \tag{5}$$

where $i=1,2,\ldots,N$ and $|M|\gg|m|$. This theory is renormalizable in $2+1$ dimensions and can, therefore, be studied in its own right. In particular, let us note that integrating out the heavy fermions will result in an effective Lagrangian of the form [9]

$$\mathscr{L}_{\mathrm{eff}}=\bar{\psi}(\mathrm{i}\partial\!\!\!/-m+gA\!\!\!/)\psi+\frac{N\tilde{g}^2}{8\pi}\frac{M}{|M|}\varepsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda-\frac{N\tilde{g}^2}{24\pi|M|}F_{\mu\nu}F^{\mu\nu}+\mathcal{O}\left(\frac{1}{M^2}\right). \tag{6}$$

It is clear, therefore, that we can think of the Chern–Simons theory of (1) as the low energy effective theory resulting from (5) with

$$\kappa=N\tilde{g}^2\frac{M}{|M|}. \tag{7}$$

Note that if we choose $\tilde{g}=1$, then statistics, in this case, would become fractional for $N\geqslant3$. (One can, in fact, go ahead and even try to argue at this point that this is an example of violation of the decoupling theorem [10], but we do not pursue this question any further in the present letter.) To study the temperature dependence of the anomalous statistics, we can, therefore, study the fermion system of (5) at finite temperature. The heavy fermions can once again be integrated out using either the imaginary time formalism [11] or the real time formalism [12] and the low energy effective theory can be most conveniently obtained from the method of derivative expansion [13]. We omit the technical details [14] and merely quote here the results of such a calculation at finite temperature:

$$\begin{aligned}\mathscr{L}_{\mathrm{eff}}=\;&\bar{\psi}(\mathrm{i}\partial\!\!\!/-m+gA\!\!\!/)\psi+\frac{N\tilde{g}^2}{8\pi}\frac{M}{|M|}\tanh\left(\frac{|M|\beta}{2}\right)\varepsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda\\&-\frac{N\tilde{g}^2}{24\pi}\frac{1}{|M|}\tanh\left(\frac{|M|\beta}{2}\right)F_{\mu\nu}F^{\mu\nu}+\mathcal{O}\left(\frac{1}{|M|^2}\right).\end{aligned} \tag{8}$$

Here $\beta = (kT)^{-1}$ and once again, it is clear that the theory at finite temperature (8) is equivalent to the Chern–Simons theory of (1) at low energy with

$$\kappa(T) = N\tilde{g}^2 \frac{M}{|M|} \tanh\left(\frac{|M|\beta}{2}\right). \tag{9}$$

We note that the coefficient of the Chern–Simons term, in the present case, has become temperature dependent. Consequently, following our earlier discussion, we conclude that angular momentum and, therefore, statistics will also become temperature dependent. Let us note that as $T \to 0$ ($\beta \to \infty$)

$$\kappa(T=0) = N\tilde{g}^2 \frac{M}{|M|} \tag{10}$$

which is consistent with (7). For any finite T, however, κ will have a non-trivial temperature dependent value. For $T \to \infty$ ($\beta \to 0$)

$$\kappa(T \to \infty) \to 0. \tag{11}$$

As we argued before, this is the case where we expect statistics to be normal. Unfortunately, such a phase transition will occur only at an unphysical temperature. One can, on the other hand, view such a result with optimism, namely, that in such a (2+1)-dimensional universe all statistics would have been normal in the beginning. Our results can be generalized to the non-relativistic case as well where the connection between the Chern–Simons term and the statistics is better understood.

We would like to thank Professor A Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Center for Theoretical Physics. One of us (AD) would like to thank Dr P Panigrahi for some useful discussion. This work was supported in part by US Department of Energy contract No DE-AC02-76ER13065.

References

[1] Hagen C R 1984 *Ann. Phys.* **157** 342
[2] MacKenzie R and Wilczek F 1988 *Int. J. Mod. Phys.* A **3** 2827
De Sousa Gerbert P 1991 *Int. J. Mod. Phys.* A **6** 173
[3] Haldane F 1983 *Phys. Rev. Lett.* **51** 605
Halperin B 1984 *Phys. Rev. Lett.* **52** 1583
Arovas D, Schrieffer R and Wilczek F 1984 *Phys. Rev. Lett.* **53** 722
Girvin S *The Quantum Hall Effect* ed R E Prange and S Girvin
Randjbar-Daemi S, Salam A and Strathdee J 1991 *Int. J. Mod. Phys.* B **5** 845
[4] Fetter A, Hanna C and Laughlin R 1989 *Phys. Rev.* B **39** 9679
Chen Y *et al* 1989 *Int. J. Mod. Phys.* B **3** 1001
Wen X and Zee A 1989 Quantum statistics and superconductivity in two spatial dimensions *Preprint* NSF-ITP-89-155
Randjbar-Daemi S, Salam A and Strathdee J 1990 *Nucl. Phys.* B **340** 403
[5] Lykken J, Sonnenschein J and Weiss N *Fermilab-Publication 91-41-T*
[6] Wen X and Zee A 1989 *J. Physique* **50** 1623
[7] Semenoff G 1988 *Phys. Rev. Lett.* **61** 517
Semenoff G and Sodano P 1989 *Nucl. Phys.* B **328** 753
[8] Hagen C R 1985 *Phys. Rev.* D **31** 2135
[9] Deser S and Redlich A N 1988 *Phys. Rev. Lett.* **61** 1541 and references therein; see also [14]
[10] Appelquist T and Carazzone J 1975 *Phys. Rev.* D **11** 2856

[11] Matsubara T 1955 *Prog. Theor. Phys.* **14** 351
[12] Umezawa H, Matsumoto H and Tachiki M 1982 *Thermo Field Dynamics and Condensed States* (Amsterdam: North-Holland)
[13] Fraser C M 1985 *Z. Phys.* C **28** 101
Novikov V A, Shifman M A, Vainshtein A I and Zakharov V I 1984 *Sov. J. Nucl. Phys.* **39** 77
Aitchison I J R and Fraser C M 1985 *Phys. Rev.* D **31** 2605
[14] Babu K S, Das A and Panigrahi P 1987 *Phys. Rev.* D **36** 3725